

Math 115

Fall 2017

Lecture 13



Graphing / Shading linear inequalities:

Dashed lines

$$x < a$$

Left

Solid lines

$$x \leq a$$

Vertical
lines

$$x > a$$

Right

$$x \geq a$$

$$y < b$$

below

$$y \leq b$$

Horizontal
line

$$y > b$$

Above

$$y \geq b$$

$$y < mx + b$$

below

$$y \leq mx + b$$

Slant
lines

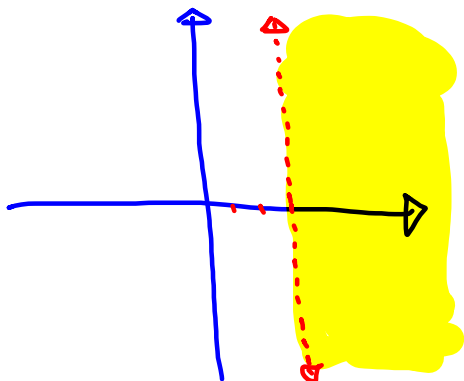
$$y > mx + b$$

Above

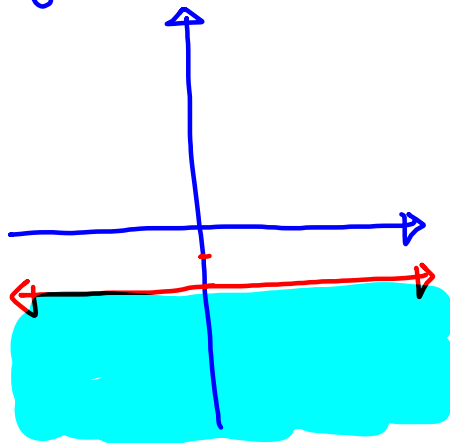
$$y \geq mx + b$$

Graph & Shade

$$x > 3$$



$$y \leq -2$$

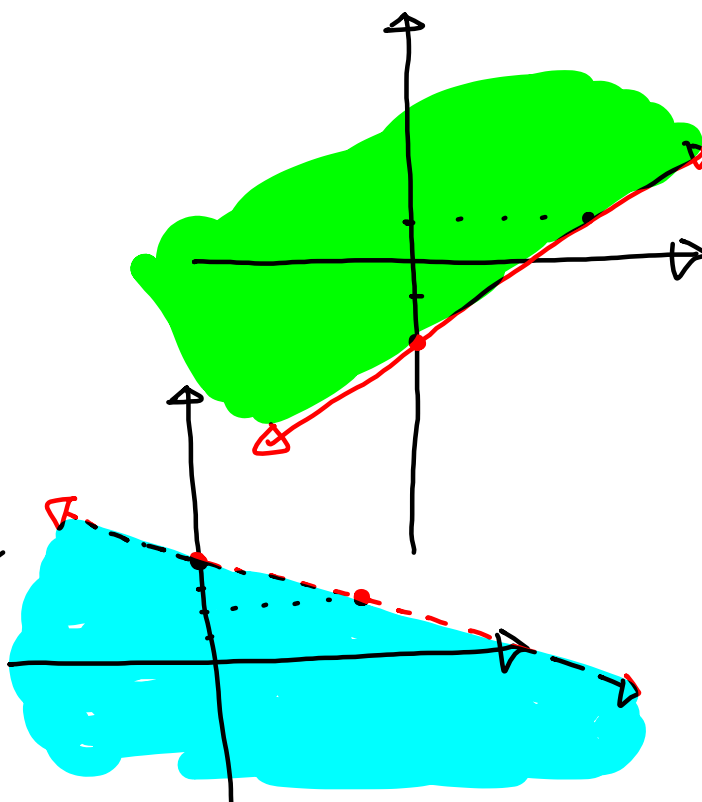


Graph & Shade

$$y \geq \frac{3}{4}x - 2$$

$$y < \frac{-2}{5}x + 4$$

Graph & Shade



Graph & Shade

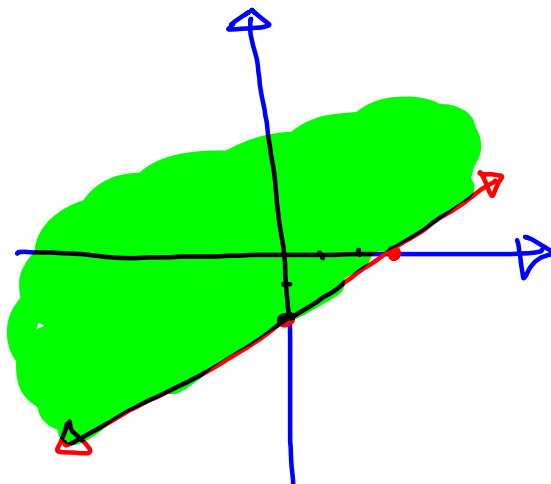
$$2x - 3y \leq 6$$

$$-3y \leq -2x + 6$$

Divide by -3

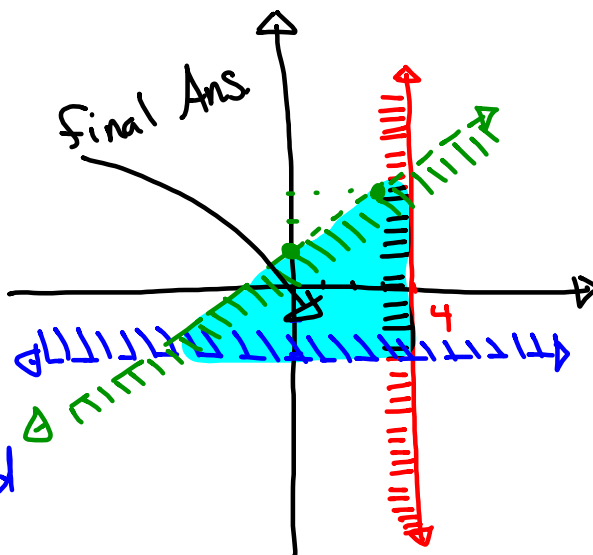
$$\frac{-3}{-3}y \geq \frac{-2}{-3}x + \frac{6}{-3}$$

$$y \geq \frac{2}{3}x - 2$$

Hint: Write in
Slope-Int Form

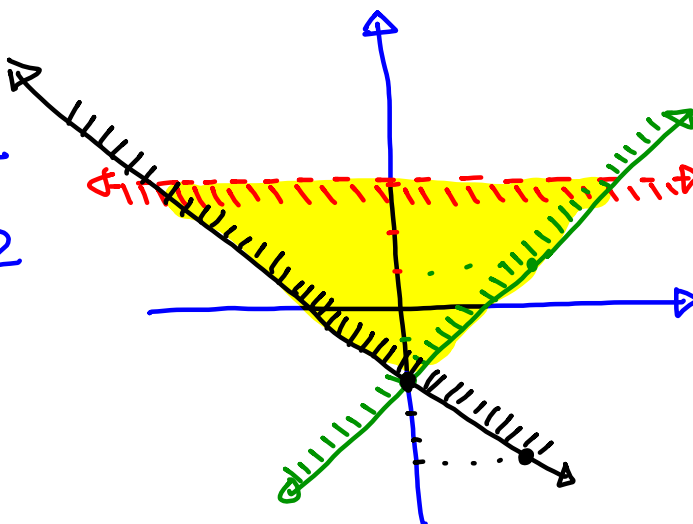
Graph & Shade

$$\begin{cases} x \leq 4 \\ y > -2 \\ y < \frac{2}{3}x + 1 \end{cases}$$

Final answer should
be the region shaded
by all Three.

Graph & Shade

$$\begin{cases} y < 3 \\ y \geq \frac{3}{4}x - 2 \\ y \geq -\frac{3}{4}x - 2 \end{cases}$$



System of linear Equations in 2-Variables:

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases}$$

A solution if exists, is
an ordered-Pair that
has to satisfy all

Is (1, 4) a

Solution for the System above?

$$x + y = 5$$

$$1 + 4 = 5 \checkmark$$

$$x - y = 1$$

$$1 - 4 = 1$$

$$-3 = 1 \text{ false}$$

(1, 4) is
not a
Solution.

Is $(3, 2)$ a Solution of

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases} \quad \begin{array}{l} 3 + 2 = 5 \\ 5 = 5 \checkmark \end{array} \quad \begin{array}{l} 3 - 2 = 1 \\ 1 = 1 \checkmark \end{array}$$

So $(3, 2)$ is a Solution of the system.

Is $(-4, 2)$ a Solution of $\begin{cases} x + 2y = 0 \\ -3x + 4y = -20 \end{cases}$?

$$\begin{array}{l} x + 2y = 0 \\ -4 + 2(2) = 0 \\ -4 + 4 = 0 \checkmark \end{array} \quad \begin{array}{l} -3x + 4y = -20 \\ -3(-4) + 4(2) = -20 \\ 12 + 8 = -20 \times \\ 20 = -20 \text{ false} \end{array} \quad \begin{array}{l} -3x + 4y = -20 \\ (-4, 2) \text{ is} \\ \text{not a} \\ \text{Soln} \end{array}$$

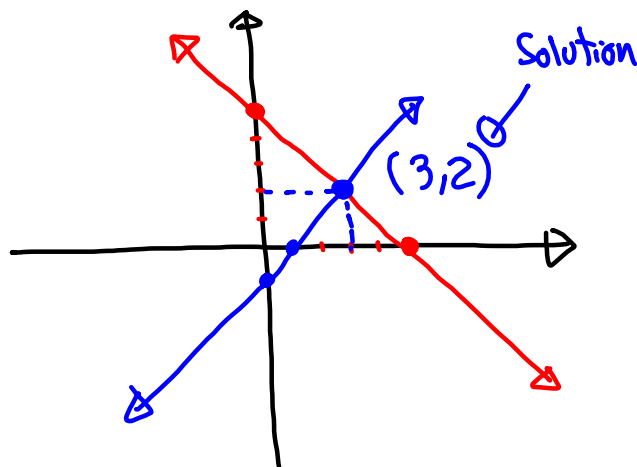
We Study 3 methods in this class on
how to Solve system of equations

1) Graphing

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases}$$

2) Substitution

3) Elimination



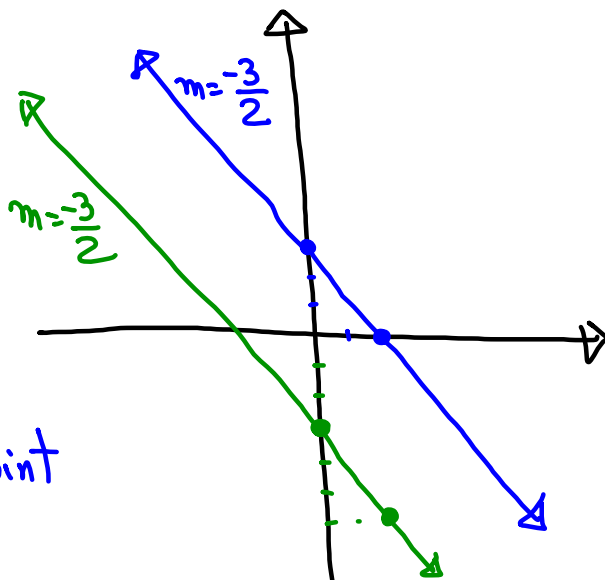
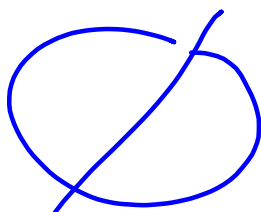
Solve by graphing

$$\begin{cases} 3x + 2y = 6 \\ y = -\frac{3}{2}x - 3 \end{cases}$$

$$y = -\frac{3}{2}x - 3$$

Parallel lines

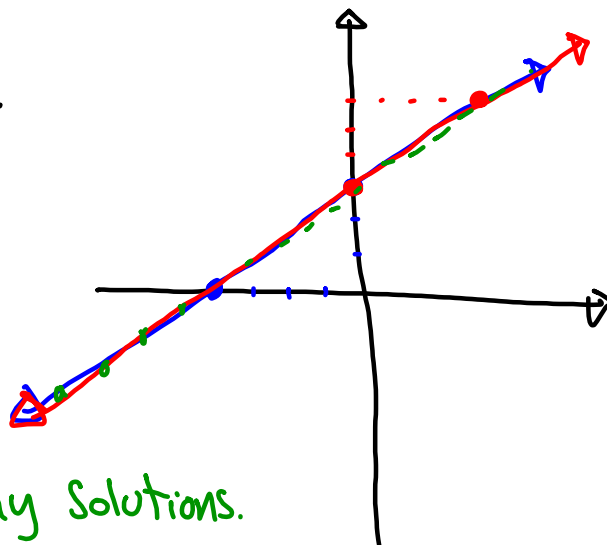
No intersection Point



Solve by graphing

$$\begin{cases} 3x - 4y = -12 \\ y = \frac{3}{4}x + 3 \end{cases}$$

$$y = \frac{3}{4}x + 3$$



Infinitely Many Solutions.

$$\begin{cases} 3x - 2y = 5 \end{cases}$$

$$\begin{cases} y = x - 4 \end{cases}$$

$$y = -3 - 4$$

$$y = -7$$

Substitution method

$$3x - 2(x - 4) = 5$$

$$3x - 2x + 8 = 5$$

$$x = 5 - 8$$

$$x = -3$$

$$(-3, -7)$$

Solve by Subs. method:

$$\begin{cases} 5x + 2y = -10 \end{cases}$$

$$\begin{cases} y = -4 - 2x \end{cases}$$

$$y = -4 - 2(-2)$$

$$= -4 + 4$$

$$y = 0$$

$$5x + 2(-4 - 2x) = -10$$

$$5x - 8 - 4x = -10$$

$$x - 8 = -10$$

$$x = -10 + 8$$

$$x = -2$$

$$(-2, 0)$$

The sum of two numbers is 5
one of them is 3 more than the
other one. Use system of linear
eqns in two variables to find them.

$$\begin{cases} x + y = 5 \\ x = y + 3 \end{cases}$$

Numbers are
4 and 1.

Use Subs. method.

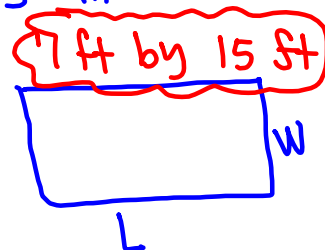
$$y + 3 + y = 5$$

$$2y = 2 \quad y = 1$$

$$x = 1 + 3 \quad x = 4$$

Perimeter of a rectangular room is 44 ft.
Its length is 1 ft more than twice its
width.

Use system of linear eqns in two variables
to find its dimensions.



$$\begin{cases} 2L + 2W = 44 \\ L = 2W + 1 \end{cases}$$

$$L = 2(7) + 1$$

$$L = 15$$

$$2(2W + 1) + 2W = 44$$

$$4W + 2 + 2W = 44$$

$$6W = 42 \quad W = 7$$

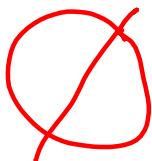
Solve by Subs.

$$\begin{cases} 6x - 3y = 5 \\ y = 2x - 3 \end{cases}$$

$$6x - 3(2x - 3) = 5$$

$$\cancel{6x} - \cancel{6x} + 9 = 5$$

$$9 = 5 \text{ false}$$



$$\begin{cases} 4x - y = 8 \\ y = 4x - 8 \end{cases}$$

$$4x - (4x - 8) = 8$$

$$4x - 4x + 8 = 8$$

$$8 = 8$$

Infinitely Many Solutions

True

Recall

$$\begin{cases} x + y = 5 \\ x - y = 1 \end{cases}$$

$$\begin{array}{r} x + y = 5 \\ x - y = 1 \\ \hline 2x = 6 \end{array}$$

$$\boxed{x=3}$$

Elimination Method

$$3 + y = 5$$

$$\boxed{y=2}$$

Final Ans (3,2)

Some books
call this
Addition
method.

Solve by Addition/Elimination method:

$$3 \begin{cases} 2x - y = -2 \\ -x + 3y = -4 \end{cases} \Rightarrow \begin{cases} 6x - 3y = -6 \\ -x + 3y = -4 \end{cases}$$

$$\begin{array}{r} 6x - 3y = -6 \\ -x + 3y = -4 \\ \hline 5x = -10 \end{array}$$

$$-(-2) + 3y = -4$$

$$\boxed{x=-2}$$

$$2 + 3y = -4$$

$$3y = -4 - 2$$

$$3y = -6$$

$$\boxed{y=-2}$$

$$(-2, -2)$$

Solve by elimination method

$$\begin{cases} x + 4y = 14 \\ 5x + 3y = 2 \end{cases} \Rightarrow \begin{cases} -5x - 20y = -70 \\ 5x + 3y = 2 \end{cases}$$

$$x + 4(4) = 14$$

$$x + 16 = 14$$

$$\boxed{x = -2}$$

$$-17y = -68$$

$$\boxed{y = 4}$$

$$(-2, 4)$$

Solve by elimination

$$\begin{cases} -2x + 3y = 10 \\ 3x + 4y = 2 \end{cases} \Rightarrow \begin{cases} -6x + 9y = 30 \\ 6x + 8y = 4 \end{cases}$$

$$3x + 4\left(\frac{2}{3}\right) = 2$$

$$17y = 34$$

$$\boxed{y = 2}$$

$$3x + 8 = 2$$

$$3x = 2 - 8$$

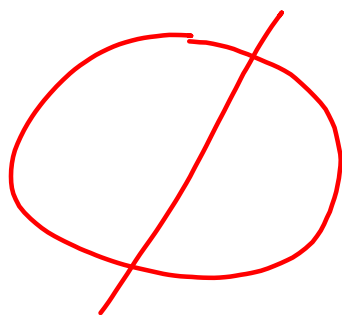
$$3x = -6$$

$$\boxed{x = -2}$$

$$(-2, 2)$$

Solve by elimination/addition:

$$\begin{aligned}
 -3 \begin{cases} 3x + y = 4 \\ 9x + 3y = -6 \end{cases} &\Rightarrow \begin{cases} -9x - 3y = -12 \\ 9x + 3y = -6 \end{cases} \\
 &\hline
 0 &= -18 \\
 &\text{false}
 \end{aligned}$$



Solve

$$\begin{aligned}
 \div 2 \begin{cases} 4x - 6y = 8 \\ 6x - 9y = 12 \end{cases} &\quad \text{even, Divisible by 2} \\
 \div 3 \begin{cases} 6x - 9y = 12 \\ 2x - 3y = 4 \end{cases} &\quad \text{Divisible by 3}
 \end{aligned}$$

$$\begin{aligned}
 \begin{cases} 2x - 3y = 4 \\ 2x - 3y = 4 \end{cases} &\Rightarrow \begin{cases} 2x - 3y = 4 \\ -2x + 3y = -4 \end{cases} \\
 -1 \begin{cases} 2x - 3y = 4 \\ 2x - 3y = 4 \end{cases} &\Rightarrow \begin{cases} 2x - 3y = 4 \\ -2x + 3y = -4 \end{cases} \\
 &\hline
 0 &= 0
 \end{aligned}$$

Infinitely Many Solns.

$0 = 0$
True

$$\begin{array}{l}
 3 \left\{ \begin{array}{l} x - \frac{y}{3} = -1 \\ \frac{-x}{2} + \frac{y}{8} = \frac{1}{4} \end{array} \right. \quad \text{LCD}=3 \quad \text{LCD}=8
 \end{array}$$

$\frac{4}{8} \cdot \frac{-x}{2} + \frac{4}{8} \cdot \frac{y}{8} = \frac{2}{4} \cdot \frac{1}{4}$

$$\begin{array}{l}
 \left\{ \begin{array}{l} 3x - y = -3 \\ -4x + y = 2 \end{array} \right. \quad \begin{array}{l} -4x + y = 2 \\ -4(1) + y = 2 \end{array} \\
 \hline
 \begin{array}{rcl}
 -x & & = -1 \\
 \boxed{x=1} & &
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 -4 + y = 2 \\
 \boxed{y=6}
 \end{array}$$

$$(1, 6)$$

Solve

Hint: Clear fractions by using LCD.

$$\begin{array}{l}
 8 \left\{ \begin{array}{l} \frac{x}{2} + \frac{y}{8} = 3 \\ x - \frac{y}{4} = 0 \end{array} \right. \quad \text{LCD}=8 \quad \text{LCD}=4
 \end{array}$$

$$\begin{array}{l}
 \left\{ \begin{array}{l} 4x + y = 24 \\ 4x - y = 0 \end{array} \right. \quad 4(3) - y = 0 \\
 \hline
 \begin{array}{rcl}
 8x & & = 24 \\
 \boxed{x=3} & &
 \end{array}
 \end{array}$$

$$\boxed{12=y}$$

$$(3, 12)$$

Solve

Hint: See last example.

$$4 \begin{cases} \frac{x}{2} + \frac{y}{4} = 1 & \text{LCD}=4 \end{cases}$$

$$8 \begin{cases} -\frac{x}{4} - \frac{y}{8} = 1 & \text{LCD}=8 \end{cases}$$

$$\begin{cases} 2x + y = 4 \\ -2x - y = 8 \end{cases}$$

$$0 = 12$$

False



Jose has \$3.80 in Dimes & Quarters.

He has 20 coins. How many of each?

$$\begin{cases} D + Q = 20 \\ 10D + 25Q = 380 \end{cases} \xrightarrow{-2} \begin{cases} D + Q = 20 \\ 2D + 5Q = 76 \end{cases}$$

$$\begin{cases} -2D - 2Q = -40 \\ 2D + 5Q = 76 \end{cases}$$

$$3Q = 36$$

$$Q = 12$$

12 Quarters
&
8 Dimes

Find eqn of a line that contains
 $(-4, 2)$ and parallel to $y = \frac{1}{2}x - 1$.

Same slope $m = \frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - (-4))$$

$$y - 2 = \frac{1}{2}(x + 4)$$

$$y - 2 = \frac{1}{2}x + 2$$

$$\boxed{y = \frac{1}{2}x + 4}$$

Find eqn of a line that contains
 $(3, -5)$ and perpendicular to $y = \boxed{-\frac{3}{4}}x + 2$.

Product of slopes is -1,

$$m = \frac{4}{3}$$

$$\frac{4}{3} \cdot -\frac{3}{4} = -1$$

$$y - -5 = \frac{4}{3}(x - 3)$$

$$y + 5 = \frac{4}{3}x - 4$$

$$\boxed{y = \frac{4}{3}x - 9}$$

Find eqn of a line that contains

$(5, 0)$ & $(0, -2)$.

$$m = \frac{0 - (-2)}{5 - 0} = \frac{2}{5}$$

$$y - (-2) = \frac{2}{5}(x - 0)$$

$$y + 2 = \frac{2}{5}x$$

Exam II Monday

Project I Monday

Be prepared to turn in

SG 9 & 10 tomorrow.

$$y = \frac{2}{5}x - 2$$